Web Appendix for “Habitual Voting and Behavioral Turnout” by James H. Fowler

Proof of Proposition 1:

Notice in equation (3) that the min condition applies to propensities above a certain cutpoint to keep them from exceeding 1. It is easy to show that this cutpoint is \( 1 - \alpha \). Thus, change in the propensity due to reinforcement is

\[
\frac{\alpha}{(1 - \alpha)} \leq p_{i,t}(L) \leq \frac{\alpha}{1 - \alpha}.
\]

Notice in equation (4) that the max condition applies to propensities below a certain cutpoint to keep them from falling below 0. It is easy to show that this cutpoint is \( \alpha \). Thus, change in the propensity due to inhibition is

\[
\frac{\alpha}{\alpha - 1} \leq p_{i,t}(L) \leq \frac{\alpha}{\alpha - 1}.
\]

Notice that the interval \( p_{i,t}(L) \in [\alpha, 1 - \alpha] \) is not empty as long as the speed of adjustment is not too large: \( \alpha / 2 \leq \alpha \). In this interval there is no feedback since the magnitudes of change due to reinforcement and inhibition are equal: \( \alpha = \alpha \).

Program to Generate Simulations Used in Text

```r
## R code for “Habitual Voting and Behavioral Turnout”
## Journal of Politics
## James Fowler
## December 8, 2003
##
## nPeriods<-1000 ## number of periods
## nSims<-1000 ## number of simulations
## nDems<-5000 ## number of Democrats
## nReps<-5000 ## number of Republicans
## n<-nDems+nReps ## number of citizens
## winPayoffD<-1.0 ## Dem payoff for winning
## winPayoffR<-1.0 ## Rep payoff for winning
## losePayoffD<-0 ## Dem payoff for winning
## losePayoffR<-0 ## Rep payoff for winning
## costD<- 0.25 ## cost to the democrats
## costR<- 0.25 ## cost to the republicans
## aspirationD<- 0.5 ## initial aspiration Dems
## aspirationR<- 0.5 ## initial aspiration Reps
## iturnoutpropensityD<-0.5 ## initial propensity to turnout Dems
## iturnoutpropensityR<-0.5 ## initial propensity to turnout Reps
##
## auxilary parameters
## tau<- 0 ## if 1, use Bush Mosteller rule, if 0, unbiased adjustment
## alpha<-0.1 ## propensity update weight for success
## beta<-0.1 ## propensity update weight for failure
## lambda<-0.95 ## weight for aspiration update
## inert<-0.01 ## probability a voter updates propensity or aspiration
## support<-0.2 ## support of random payoff shock
##
## payoff function
## payofff<-function(winner,preference,cost,action)
## preference*(winner*winPayoffR+(1-winner)*losePayoffR)+
## (1-preference)*(winner*losePayoffD+(1-winner)*winPayoffD)-
## action*cost+round(runif(length(action),-support/2,support/2),3)
##
## aspiration update function
## aspirationf<-function(aspiration,payoff)
## ((aspiration*payoff)*floor(1000*(lambda*aspiration+(1-lambda)*payoff)))+
## (aspiration*payoff)*ceiling(1000*(lambda*aspiration+(1-lambda)*payoff)))/1000+
## (aspiration*payoff)*aspiration
```

## adjustment for propensities

```r
propensityf <- function(propensity, aspiration, action, payoff) {
  pmin(1, pmax(0, {
    (payoff > aspiration) * ceiling(1000 * (propensity + alpha * (1 - tau * propensity))) +
    (payoff < aspiration) * floor(1000 * (propensity - beta * (1 - tau * (1 - propensity)))) +
    (1 - action) *
    ((payoff > aspiration) * floor(1000 * (propensity - alpha * (1 - tau * (1 - propensity)))) +
     (payoff < aspiration) * ceiling(1000 * (propensity + beta * (1 - tau * propensity)))) / 1000)
}
```

## voter preferences

```r
preferences <- c(rep(0, nDems), rep(1, nReps))
```

## voter costs

```r
costs <- c(rep(costD, nDems), rep(costR, nReps))
```

```r
for (i in 1:nSims) {
  ## initial values
  voteDts <- NULL  ## init Dem vote time series
  voteRts <- NULL  ## init Dem vote time series

  ## propensity to vote
  propensities <- c(rep(iturnoutpropensityD, nDems), rep(iturnoutpropensityR, nReps))

  ## voter aspirations
  aspirations <- c(rep(iaspirationD, nDems), rep(iaspirationR, nReps))

  for (j in 1:nPeriods) {
    ## voters choose whether or not to vote
    actions <- runif(n, 0, 1) < propensities

    ## total votes
    voteD <- sum((1 - preferences) * actions)
    voteR <- sum(preferences * actions)
    voteDts <- c(voteDts, voteD / nDems)
    voteRts <- c(voteRts, voteR / nReps)

    ## winner determined
    winner <- (voteR > voteD) + (voteD == voteR) * (runif(1, 0, 1) > 0.5)

    ## voter payoffs
    payoffs <- payofff(winner, preferences, costs, actions)

    ## set up voter index vector for propensities
    noninertials <- which(runif(n, 0, 1) > inert)
    propensities[noninertials] <- propensityf(propensities[noninertials], aspirations[noninertials], actions[noninertials], payoffs[noninertials])

    ## set up voter index vector for aspirations
    noninertials <- which(runif(n, 0, 1) > inert)
    aspirations[noninertials] <- aspirationf(aspirations[noninertials], preferences[noninertials])
  }
}
Over the course of developing this article, I benefited greatly from comments and advice by Jonathan Bendor. John Geer suggested that I outline some of our remaining points of disagreement and include them in this web appendix.

Cognitive Psychology and Behavioral Turnout

Bendor argues that modern cognitive psychology will reject both the BDT model and the alternative model because they are too “impoverished” and their psychology too “sparse.” While I cannot speak for cognitive psychologists in particular, I can say that if we can build a model that accurately predicts and explains behavior on a number of levels, then our efforts to formalize behavioralism are more likely to be well-received. The paper gives a nod to the fact that the decline of behaviorist models in the 1970s is a complex story, but both Camerer and Diaconis and Lehman (cited in the text) specifically isolate the failure to predict individual level behavior as particularly important.

“Always voting” and “never voting” in the model

Bendor shows that \( p=0 \) and \( p=1 \) are not absorbing states in the alternative model and argues that Bush Mosteller and the alternative model are the same at the extremes. But this critique is irrelevant. In fact, Proposition 1 already points this out by noting that there is no moderating feedback in the range \( p_{i,v}(I) \in [\alpha, 1 - \alpha] \). Obviously, since \( \alpha > 0 \), the alternative model will experience moderating feedback at the extremes because of ceiling effects, a point on which I elaborate in the article. Bendor infers from this fact that “always voting” and “never voting” are not possible in the model. There are three problems with this argument. First, it is not technically true. One can have a propensity to vote of only 0.9 and still vote in every election for some finite series, depending on the random draws (e.g. the probability of “always voting” in a sequence of 3 elections is about 0.7 if the propensity to vote remains constant at 0.9). Second, the main point is that the propensities to vote in the Bush Mosteller model are bunched up around 0.5 while those in the alternative model are more likely to be near (if not exactly at) the extremes. Thus the alternative model will generate a lot more individuals engaging in the same behavior for a long string of elections. Third, Bendor’s own computer program for generating simulations shows that the distribution of propensities using the alternative model has much higher variance than the Bush Mosteller model with many more individuals at or near propensities of 0 and 1. Thus, this argument about propensities at the extremes does not invalidate the analytical arguments I make and the simulation evidence I bring to bear that the alternative model generates more habitual behavior than the BDT model.
Propositions in BDT Do Not Apply to the Alternative Model

Bendor argues in his review of the paper that the alternative model is covered by BDT’s more general model in propositions 1-3 in the original paper, thus if BDT’s “analytical results explain why their benchmark computational model generates appreciable aggregate turnout, the present MS’s claim that these computational results are being driven by special features of Bush-Mosteller is undermined.”

This argument is false. Proposition 1 in their paper relates only to ergodicity while Proposition 2 establishes how many individuals will change their voting propensities (not actual behavior) in the next time period. These propositions are not sufficient to make aggregate level predictions about expected turnout without knowing the functional form of the reinforcement mechanism because they say nothing about how much the propensities will change.

Proposition 3 does say something about this (and proposition 5) but these propositions do not apply to the alternative model. As I explain in the text, propositions 3 and 5 in BDT suggest that if voters use an aspiration-based adjustment rule like Bush Mosteller or the alternative rule presented in my article, the average propensity to vote will increase when all voters have propensities less than or equal to 0.5 and decrease when all voters have propensities greater than or equal to 0.5. However, since any \( p_{i,t} \in [\alpha, 1 - \alpha] \) can be stable, there is almost always at least one voter with a propensity above 0.5 and one with a propensity below 0.5. In fact, in 100,000 simulations with randomly drawn parameters the conditions for proposition 3 and 5 were not met even once. The limiting distribution always had some individual propensities above and some below 0.5.

Stationary vs. Non-Stationary Models

Finally, Bendor suggests that I develop a non-stationary model instead of a stationary one, since it might help to explain how people gradually become fixated on a certain probability of voting. This is an excellent idea, and I plan to explore it in the future. However, since the current stationary model yields habitual behavior without extra assumptions, it seems reasonable to explore it first.