Ideology and Social Networks in the U.S. Congress

James H. Fowler
University of California, San Diego

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- Improve "connectedness scores" (Fowler 2005)
- Cosponsorship is about 2 things:
  - The idea
  - The person behind the idea
- The goal is to create a Social Space Model that simultaneously estimates ideological and social ideal points.
  - Place legislators and bills in policy space (Poole and Rosenthal 1997; Clinton, Jackman, and Rivers 2004)
  - Place sponsors and cosponsors in social space (Hoff, Raftery, and Handcock 2002)
- Create alternative Social Utility Model

- n legislators choose whether to cosponsor m bills.
- Each bill $j = 1 \ldots m$ is sponsored by legislator $s(j)$
- Each legislator $i = 1 \ldots n$ chooses between the bill at ζ_j and the status quo located at ψ_j, both in $\mathbf{R}^d$

$$y_{ij} = \begin{cases} 1 & \text{if legislator } i \text{ cosponosors bill } j \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

- Legislator i receives utility for supporting *policies* close to her own ideal point $\mathbf{x}_i$ in $\mathbf{R}^d$ *policy* space:

$$\begin{aligned} U_{ij}^{bill} &= -\|\mathbf{x}_i - \zeta_j\|^2 \\ U_{ij}^{status\ quo} &= -\|\mathbf{x}_i - \psi_j\|^2 \\ U_{ij}^{cosponsor} &= -\|\mathbf{x}_i - \zeta_j\|^2 + \|\mathbf{x}_i - \psi_j\|^2 \end{aligned} \tag{2}$$

- Legislator i also receives utility for supporting *people* close to her own *social* ideal point in $\mathbf{R}^{\delta}$ social space
  - $\boldsymbol{\lambda}_i$ = legislator i's ideal point in $\mathbf{R}^{\delta}$ social *sender* space (or *hub* space, Kleinberg 1999)
  - $\boldsymbol{\alpha}_{s(j)}$ = the ideal point for bill j's sponsor in $\mathbf{R}^{\delta}$ social *receiver* space (or *authority* space)

$$
\begin{aligned}
U_{ij}^{social} &= -\|\boldsymbol{\lambda}_i - \boldsymbol{\alpha}_{s(j)}\|^2 \\
U_{ij}^{ideological} &= -\|\mathbf{x}_i - \mathbf{x}_{s(j)}\|^2 \\
U_{ij}^{personal} &= -\|\boldsymbol{\lambda}_i - \boldsymbol{\alpha}_{s(j)}\|^2 - \kappa\|\mathbf{x}_i - \mathbf{x}_{s(j)}\|^2 \qquad (3)
\end{aligned}
$$

- Utility also affected by bill-specific factors ϕ_j and legislator-specific factors ξ_i:

$$U_{ij} = -\|\mathbf{x}_i - \boldsymbol{\zeta}_j\|^2 + \|\mathbf{x}_i - \boldsymbol{\psi}_j\|^2 - \|\boldsymbol{\lambda}_i - \boldsymbol{\alpha}_{s(j)}\|^2 - \kappa\|\mathbf{x}_i - \mathbf{x}_{s(j)}\|^2 + \xi_i + \phi_j \quad (4)$$

- This simplifies to

$$U_{ij} = -2\mathbf{x}_i\boldsymbol{\beta}_j + 2\boldsymbol{\lambda}_i\boldsymbol{\alpha}_{s(j)} + \eta_i + \theta_j \quad (5)$$

where

$$\boldsymbol{\beta}_j = \boldsymbol{\psi}_j - \boldsymbol{\zeta}_j - \kappa\mathbf{x}_{s(j)}, \ \eta_i = -\boldsymbol{\lambda}_i^2 - \kappa\mathbf{x}_i^2 + \xi_i$$
$$\theta_j = \boldsymbol{\psi}_j^2 - \boldsymbol{\zeta}_j^2 - \mathbf{x}_{s(j)}^2 - \kappa\mathbf{x}_{s(j)}^2 + \phi_j$$

- Suppose observed cosponsorship means true utility of cosponsorship is high (1), while not observing one means that the true utility is low (0). True utility is observed cosponsorship decision minus an error term:

$$U_{ij} = y_{ij} - \nu_{ij} \tag{6}$$

- Substituting, we get:

$$y_{ij} = -2\mathbf{x}_i\boldsymbol{\beta}_j + 2\boldsymbol{\lambda}_i\boldsymbol{\alpha}_{s(j)} + \eta_i + \theta_j + \nu_{ij} \tag{7}$$

- Define the *double-center operator* $D(.)$ for a matrix $\mathbf{Z}$ (Poole 2005; Clinton, Jackman, Rivers 2005):

$$D(z_{ij}) = (z_{ij} - \bar{z}_{i.} - \bar{z}_{.j} + \bar{z}_{..})/(-2) \tag{8}$$

- Suppose ν_{ij} is i.i.d. stable density and dimension-by-dimension means of $\mathbf{x}$, $\boldsymbol{\beta}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\alpha}_{s(j)}$ equal 0.
- Apply double-center operator in equation (8) to both sides of equation (7):

$$D(y_{ij}) = \mathbf{x}_i\boldsymbol{\beta}_j - \boldsymbol{\lambda}_i\boldsymbol{\alpha}_{s(j)} + \epsilon_{ij} \tag{9}$$

- By construction, social distance ($\lambda_i \alpha_{s(j)}$) is uncorrelated with ideological distance ($\mathbf{x}_i \beta_j$).
- Therefore, singular value decomposition (SVD) provides the best d dimensional approximation of $\mathbf{x}_i$ and β_j (Frobenius-Perron):

$$D(\mathbf{Y}) = \mathbf{X}\Sigma\mathbf{B} \tag{10}$$

- Suppose each legislator k sponsors n_k bills. We can define an n by n matrix such that:

$$\mu_{ik} = \frac{1}{n_k} \sum_{s(j)=k} D(y_{ij}) = \mathbf{x}_i \bar{\boldsymbol{\beta}}_k - \boldsymbol{\lambda}_i \boldsymbol{\alpha}_k + \upsilon_{ik} \tag{11}$$

  where $\bar{\boldsymbol{\beta}}_k = \frac{1}{n_k} \sum_{s(j)=k} \boldsymbol{\beta}_j$ and υ_{ik} is a stable density (but not i.i.d. across sponsors).
- Rearranging yields another SVD formulation:

$$\mathbf{X}\boldsymbol{\Sigma}\mathbf{B} - \mathbf{M} = \boldsymbol{\Lambda}\mathbf{T}\mathbf{A} \tag{12}$$

- Standard errors via Metropolis-Hastings!
- If errors ϵ_{ij} in equation (9) are normally distributed, then:

$$L(\mathbf{x}_i, \boldsymbol{\beta}_j, \boldsymbol{\lambda}_i, \boldsymbol{\alpha}_{s(j)} | y_{ij}) = - \sum_{i,j,(i \neq j)} \left(D(y_{ij}) - \mathbf{x}_i \boldsymbol{\beta}_j + \boldsymbol{\lambda}_i \boldsymbol{\alpha}_{s(j)} \right)^2 \tag{13}$$

- Estimate dyad-specific mean utility transfers γ_{ik} from legislator k to legislator i.
- These utilities can be thought of as
  - social influence
  - vote-trading
  - specific instances of monetary transfers from one legislator to another (such as PAC contributions to a friend's campaign)
- To control for legislative activity, assume $\bar{\gamma}_{i.} = \bar{\gamma}_{.s(j)} = \bar{\gamma}_{..} = 0$.

- Personal utility from equation (3) becomes:

$$U_{ij}^{personal} = \gamma_{is(j)} - \kappa \|\mathbf{x}_i - \mathbf{x}_{s(j)}\|^2 \tag{14}$$

- The full utility equation from (4) becomes:

$$U_{ij} = -\|\mathbf{x}_i - \boldsymbol{\zeta}_j\|^2 + \|\mathbf{x}_i - \boldsymbol{\psi}_j\|^2 + \gamma_{is(j)} - \kappa \|\mathbf{x}_i - \mathbf{x}_{s(j)}\|^2 + \xi_i + \phi_j \tag{15}$$

which simplifies to:

$$U_{ij} = -2\mathbf{x}_i \boldsymbol{\beta}_j + \gamma_{is(j)} + \eta_i + \theta_j \tag{16}$$

- Establishing a relationship between cosponsorship and the utility it provides yields an analogue to equation (7):

$$y_{ij} = -2\mathbf{x}_i\boldsymbol{\beta}_j + \gamma_{is(j)} + \eta_i + \theta_j + \nu_{ij} \tag{17}$$

- Double-centering both sides yields:

$$D(y_{ij}) = \mathbf{x}_i\boldsymbol{\beta}_j - \frac{\gamma_{is(j)}}{2} + \epsilon_{ij} \tag{18}$$

- The SVD provides the best d dimensional approximation of $\mathbf{x}_i$ and β_j:

$$D(\mathbf{Y}) = \mathbf{X}\mathbf{\Sigma}\mathbf{B} \tag{19}$$

- Suppose each legislator k sponsors n_k bills. We can define an n by n matrix such that:

$$\mu_{ik} = \frac{1}{n_k} \sum_{s(j)=k} D(y_{ij}) = \mathbf{x}_i \bar{\beta}_k - \frac{\gamma_{ik}}{2} \tag{20}$$

- Rearranging yields the social utility matrix:

$$\mathbf{\Gamma} = 2(\mathbf{X}\mathbf{\Sigma}\mathbf{B} - \mathbf{M}) \tag{21}$$

- Standard errors via Metropolis-Hastings!

$$L(\mathbf{x}_i, \beta_j | y_{ij}) = -\sum_{i,j} (D(y_{ij}) - \mathbf{x}_i \beta_j)^2 \tag{22}$$

- Generate "true" data with independent random draw from a uniform distribution for each
  - legislator (x)
  - bill (ζ)
  - status quo (ψ)

  and either:
  - legislator's sender space (λ) and receiver space (α) for Social Space Model
  - legislator dyad's specific social utility (γ) for Social Utility Model

- Parameterize social weight:

$$U_{ij}^{total} = \omega U_{ij}^{social} + (1 - \omega) U_{ij}^{ideological} \tag{23}$$

- Normally-disributed error term
- Choose a constant threshold U^* to generate specific rate of cosponsorship such that

$$y_{ij} = \begin{cases} 1 & \text{if } U_{ij} > U^* \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

- Option to change the utilities from quadratic $-u^2$ to gaussian $\sigma\phi(u)$.

- Baseline configuration:
  - 100 legislators, each sponsors on average 10 bills (for 1000 total bills)
  - rate of cosponsorship is set to 0.1 (Fowler 2005)
  - the effect of social utility is approximately equal to ideological utility ($\omega = 0.5$)
  - no error in the utility
  - utilities are quadratic
- Use Spearman rank correlation (doesn't matter).

	Legislator Ideal Point		Bill Parameter		Ideological Utility		Social Utility
	ρ	$\rho - \rho_{NOM}$	ρ	$\rho - \rho_{NOM}$	ρ	$\rho - \rho_{NOM}$	$\rho_{\lambda' \alpha}$
Baseline	0.980 0.972,0.987	-0.001 -0.004,0.001	0.948 0.938,0.956	0.105 0.092,0.122	0.868 0.841,0.890	0.164 0.144,0.187	0.795 0.740,0.843
5 Bills per Legislator	0.976 0.966,0.983	-0.001 -0.005,0.003	0.950 0.944,0.956	0.108 0.088,0.132	0.873 0.851,0.891	0.162 0.137,0.188	0.781 0.732,0.819
1 Bill per Legislator	0.956 0.935,0.970	0.002 -0.010,0.013	0.944 0.923,0.959	0.115 0.067,0.155	0.840 0.797,0.882	0.158 0.122,0.207	0.617 0.417,0.741
500 Legislators (1 Bill Each)	0.981 0.977,0.985	-0.001 -0.004,0.002	0.973 0.969,0.977	0.091 0.079,0.104	0.903 0.889,0.919	0.150 0.128,0.172	0.750 0.707,0.773
0.2 Social Weight	0.995 0.992,0.997	0.000 -0.001,0.000	0.960 0.847,0.895	0.061 0.072,0.096	0.876 -0.805,0.923	0.084 0.559,0.951	0.646 0.247,0.782
0.0 Social Weight	0.998 0.997,0.999	0.000 -0.001,0.000	0.954 0.948,0.960	0.051 0.037,0.066	0.855 0.823,0.876	0.064 0.051,0.082	0.000 -0.019,0.022

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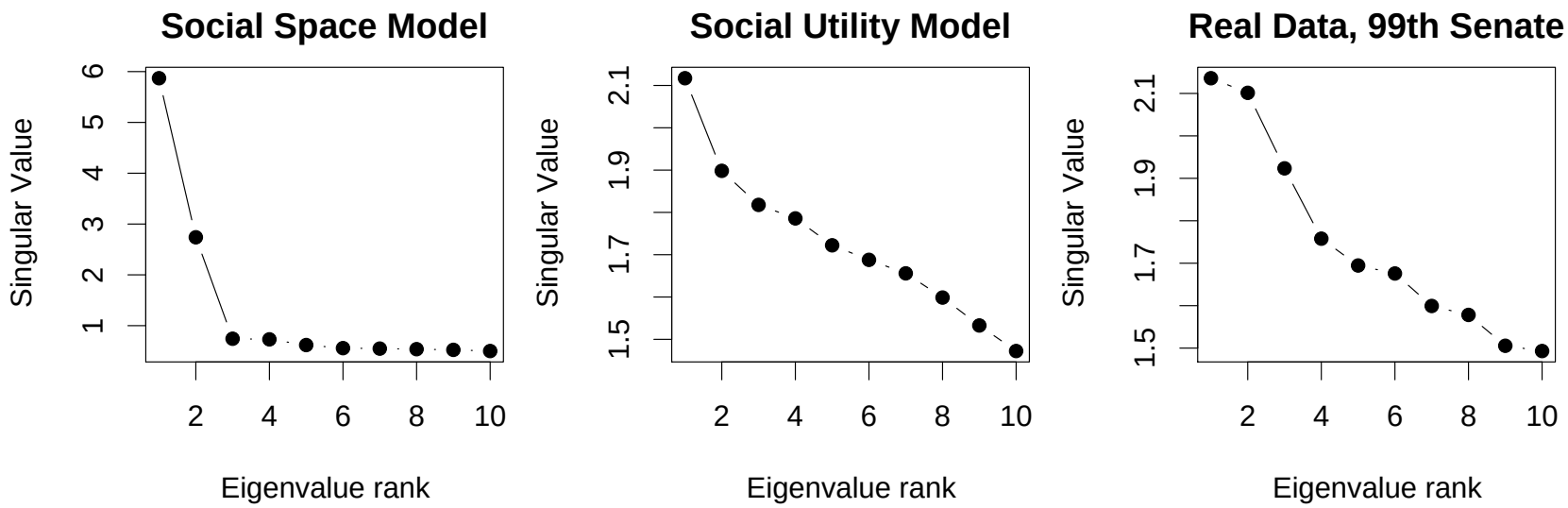
Baseline	0.980 0.972,0.987	-0.001 -0.004,0.001	0.948 0.938,0.956	0.105 0.092,0.122	0.868 0.841,0.890	0.164 0.144,0.187	0.795 0.740,0.843
0.05 Rate of Cosponsorship	0.983 0.974,0.989	-0.001 -0.003,0.000	0.923 0.912,0.932	0.094 0.075,0.113	0.810 0.762,0.846	0.191 0.153,0.244	0.757 0.708,0.802
0.2 Rate of Cosponsorship	0.975 0.965,0.986	0.001 -0.001,0.007	0.960 0.945,0.968	0.041 0.026,0.054	0.886 0.869,0.900	0.130 0.108,0.151	0.813 0.764,0.855
0.5 Standard Deviation Error	0.980 0.972,0.986	-0.001 -0.005,0.002	0.949 0.941,0.956	0.109 0.095,0.123	0.872 0.842,0.892	0.166 0.136,0.186	0.802 0.764,0.842
1.0 Standard Deviation Error	0.980 0.968,0.987	-0.002 -0.005,0.002	0.950 0.943,0.955	0.110 0.101,0.119	0.875 0.855,0.890	0.169 0.145,0.189	0.798 0.751,0.843
Gaussian Utility	0.896 0.871,0.921	-0.023 -0.038,-0.008	0.932 0.896,0.954	0.141 0.103,0.171	0.833 0.769,0.873	0.182 0.123,0.219	0.785 0.724,0.829

	Legislator Ideal Point		Bill Parameter		Ideological Utility		Social Utility
	ρ	$\rho - \rho_{NOM}$	ρ	$\rho - \rho_{NOM}$	ρ	$\rho - \rho_{NOM}$	ρ_{γ}
Baseline	0.995 0.993,0.996	0.001 0.000,0.003	0.953 0.949,0.958	0.116 0.102,0.131	0.878 0.857,0.891	0.210 0.184,0.236	0.711 0.698,0.721
5 Bills per Legislator	0.990 0.987,0.994	0.001 -0.002,0.005	0.953 0.944,0.958	0.117 0.099,0.135	0.871 0.852,0.890	0.206 0.180,0.236	0.565 0.546,0.580
1 Bill per Legislator	0.959 0.947,0.969	0.005 -0.005,0.014	0.945 0.927,0.957	0.128 0.077,0.185	0.839 0.788,0.880	0.177 0.124,0.229	0.234 0.220,0.247
500 Legislators (1 Bill Each)	0.992 0.991,0.993	0.001 0.000,0.002	0.980 0.978,0.983	0.097 0.086,0.111	0.910 0.898,0.921	0.199 0.174,0.227	0.237 0.232,0.243
0.2 Social Weight	0.998 0.997,0.998	0.000 -0.001,0.000	0.962 0.957,0.966	0.063 0.049,0.075	0.879 0.865,0.893	0.090 0.074,0.110	0.382 0.363,0.399
0.0 Social Weight	0.998 0.997,0.999	0.000 0.000,0.000	0.955 0.949,0.959	0.044 0.027,0.062	0.854 0.832,0.872	0.063 0.044,0.085	0.001 -0.016,0.020

	Legislator Ideal Point		Bill Parameter		Ideological Utility		Social Utility
	ρ	$\rho - \rho_{NOM}$	ρ	$\rho - \rho_{NOM}$	ρ	$\rho - \rho_{NOM}$	ρ_{γ}
Baseline	0.995 0.993,0.996	0.001 0.000,0.003	0.953 0.949,0.958	0.116 0.102,0.131	0.878 0.857,0.891	0.210 0.184,0.236	0.711 0.698,0.721
0.05 Rate of Cosponsorship	0.992 0.990,0.995	0.001 -0.001,0.002	0.925 0.917,0.932	0.093 0.079,0.113	0.834 0.801,0.857	0.241 0.199,0.284	0.562 0.547,0.572
0.2 Rate of Cosponsorship	0.995 0.993,0.997	0.000 0.000,0.002	0.967 0.965,0.971	0.054 0.043,0.061	0.895 0.879,0.908	0.173 0.159,0.190	0.823 0.815,0.829
0.5 Standard Deviation Error	0.994 0.991,0.996	0.001 0.000,0.002	0.954 0.950,0.958	0.116 0.104,0.129	0.880 0.859,0.898	0.206 0.181,0.238	0.710 0.699,0.723
1.0 Standard Deviation Error	0.994 0.990,0.996	0.001 -0.001,0.002	0.953 0.948,0.957	0.114 0.101,0.126	0.875 0.847,0.893	0.207 0.181,0.235	0.709 0.700,0.721
Gaussian Utility	0.929 0.910,0.941	-0.018 -0.028,-0.010	0.959 0.955,0.963	0.136 0.118,0.159	0.876 0.855,0.893	0.232 0.210,0.268	0.725 0.716,0.732

		Social Space Model			Social Utility Model		
Congress	Years	ρ_x	ρ_β	$\rho_{\boldsymbol{\lambda}'\boldsymbol{\alpha}}$	ρ_x	ρ_β	ρ_γ
93rd	1973-1974	0.988 0.983,0.992	0.941 0.936,0.946	0.772 0.741,0.803	0.996 0.989,0.998	0.935 0.898,0.956	0.724 0.453,0.896
94th	1975-1976	0.987 0.979,0.992	0.940 0.933,0.945	0.762 0.718,0.804	0.996 0.986,0.998	0.934 0.891,0.956	0.710 0.402,0.897
95th	1977-1978	0.987 0.980,0.992	0.941 0.934,0.946	0.767 0.724,0.798	0.991 0.985,0.998	0.936 0.902,0.966	0.727 0.430,0.927
96th	1979-1980	0.988 0.983,0.993	0.941 0.936,0.946	0.768 0.720,0.807	0.996 0.991,0.998	0.935 0.908,0.959	0.705 0.456,0.905
97th	1981-1982	0.989 0.986,0.993	0.939 0.933,0.946	0.767 0.718,0.802	0.997 0.986,0.999	0.932 0.886,0.961	0.754 0.460,0.942
98th	1983-1984	0.989 0.984,0.992	0.940 0.934,0.944	0.769 0.736,0.794	0.997 0.995,0.999	0.932 0.906,0.959	0.766 0.596,0.933
99th	1985-1986	0.987 0.980,0.992	0.943 0.933,0.947	0.783 0.742,0.822	0.998 0.994,0.999	0.941 0.912,0.961	0.805 0.601,0.928
100th	1987-1988	0.987 0.979,0.991	0.944 0.940,0.950	0.786 0.741,0.822	0.998 0.994,0.999	0.941 0.913,0.962	0.812 0.569,0.933

		Social Space Model			Social Utility Model		
Congress	Years	ρ_x	ρ_β	$\rho_{\boldsymbol{\lambda}'\boldsymbol{\alpha}}$	ρ_x	ρ_β	ρ_γ
101st	1989-1990	0.987 0.979,0.991	0.944 0.938,0.948	0.795 0.751,0.833	0.997 0.985,0.999	0.941 0.871,0.963	0.816 0.428,0.939
102nd	1991-1992	0.988 0.984,0.993	0.944 0.937,0.949	0.788 0.741,0.820	0.998 0.996,0.999	0.944 0.924,0.965	0.818 0.647,0.941
103rd	1993-1994	0.986 0.979,0.991	0.942 0.936,0.947	0.781 0.734,0.815	0.998 0.995,0.999	0.942 0.928,0.958	0.794 0.602,0.912
104th	1995-1996	0.989 0.983,0.993	0.940 0.935,0.945	0.763 0.713,0.796	0.982 0.970,0.999	0.920 0.862,0.956	0.724 0.341,0.911
105th	1997-1998	0.988 0.983,0.993	0.941 0.933,0.946	0.771 0.733,0.797	0.997 0.993,0.999	0.937 0.913,0.955	0.766 0.559,0.906
106th	1999-2000	0.989 0.983,0.992	0.943 0.936,0.946	0.775 0.741,0.819	0.997 0.989,0.999	0.936 0.885,0.956	0.776 0.454,0.911
107th	2001-2002	0.988 0.983,0.993	0.941 0.931,0.947	0.766 0.726,0.812	0.997 0.993,0.999	0.935 0.904,0.951	0.771 0.565,0.897
108th	2003-2004	0.987 0.979,0.992	0.942 0.934,0.947	0.775 0.718,0.816	0.997 0.987,0.999	0.936 0.903,0.960	0.780 0.523,0.925



- The Social Utility Model is Superior for Cosponsorship
  - Outlier values in left panel indicate two structured dimensions in data generated by the Social Space Model
  - Center panel shows lack of structure in data generated by the Social Utility Model
  - Right panel shows a lack of structure in data taken from the 99th Senate

- We can recover social utilities without ideology!
- SVD procedure (surprisingly) works better than W-NOMINATE
- Social Utility Model works best on real data
- Can generalize to simultaneous modeling of any bipartite network with a unipartite projection.